Conceptual design of fill dams for pump-storage reservoirs

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1 INTRODUCTION

Designing a pump storage plant involves several steps, and it requires a multidisciplinary approach that includes civil, electrical, and mechanical engineering, as well as environmental considerations. Increasingly, potential investors for pumped storage power plants expect comprehensive statements in the very first planning phase (feasibility study) regarding the best possible locations, lowest costs, cost comparisons, etc., before they can decide to commission further planning phases.

The ***ReservoirDam*** program was developed to meet these expectations despite the usually relatively low planning fee, as reservoirs in particular contribute a considerable proportion to the overall construction costs and at the same time the reservoir construction costs are heavily dependent on the shape of the terrain.

ReservoirDam is a software based on Visual Studio and AutoCAD offering an easy and rapid optimization of fill dam reservoirs regarding

- dam crown elevation (mass balance between cut and fill material)
- X and Y coordinates of center of dam crown polyline
- height of live water body
- optimization of shape of dam crown polyline (ratio between major and minor axis length A/B as defined in Figure 2-2 to Figure 2-4 below) and orientation angle AL with the aim of showing alternatives with the lowest possible construction costs for cut stripping, fill stripping, cut, fill, water-proof slope and floor face

Following data are required for optimization of dam elevation only:

- volume of live water body
- estimated upstream dam slope TAW, downstream dam slope TAA and rock slope TAR (above dam crown)
- estimation of layer thicknesses for cut stripping TCS and fill stripping TFS
- estimation of thickness for waterproof slope face TSF and thickness for waterproof floor face TFF (including filter)
- a terrain 3dSolid drawing (for example derived from contour lines generated by use of the software Civil3D)

The following additional data are required in order to minimize construction costs:

- estimation of the percentage of excavated material (cut stripping, fill stripping and cut material) that can be re-used (treated or untreated) for fill dam construction
- unit prices for cut stripping, fill stripping, cut, fill (e.g. EUR/m^3 each), water-proof slope and floor face (e.g. EUR/m^2 each)

2 NOTATION

2.1 Cross Section

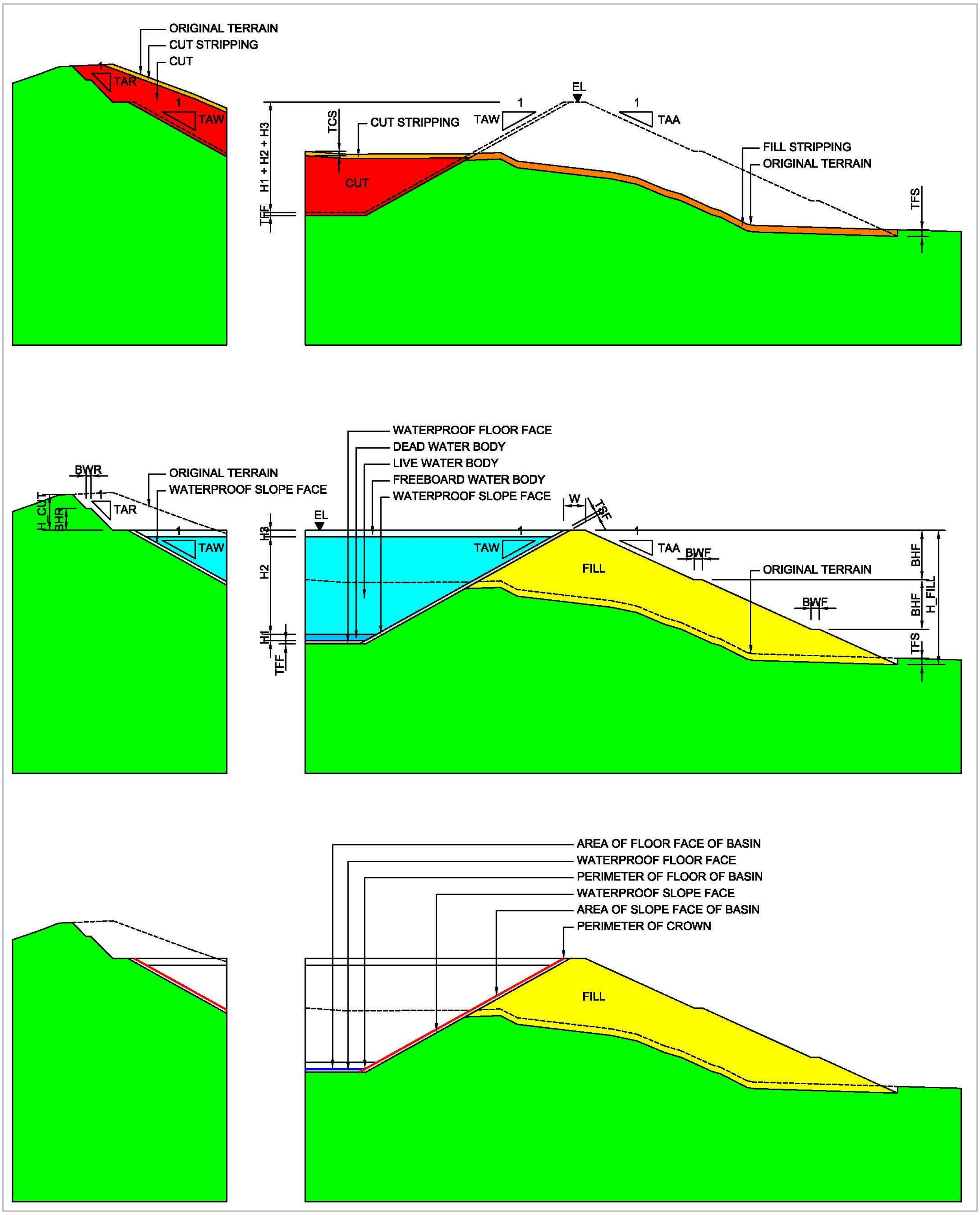


Figure 2-1: Notation – Cross Section

Legend:

W ... Crown width
EL ... Crown elevation
H1 ... Depth of dead water body
H2 ... Depth of live water body
H3 ... Freeboard
BWF ... Berm width at fill dam
BHF ... Berm height at fill dam
BWR ... Berm width at rock (above crown elevation)
BHR ... Berm height at rock (above crown elevation)
TFS ... Stripping depth below fill dam
TCS ... Stripping depth on top of cut
TSF ... Thickness of waterproof slope face (including filter)
TFF ... Thickness of waterproof floor face (including filter)
TAR ... Tangent of rock slope (above crown)
TAA ... Tangent of dam slope at air face
TAW ... Tangent of dam slope as water face

2.2 Crown Shape Types

2.2.1 Elliptically Shaped Dam Crown

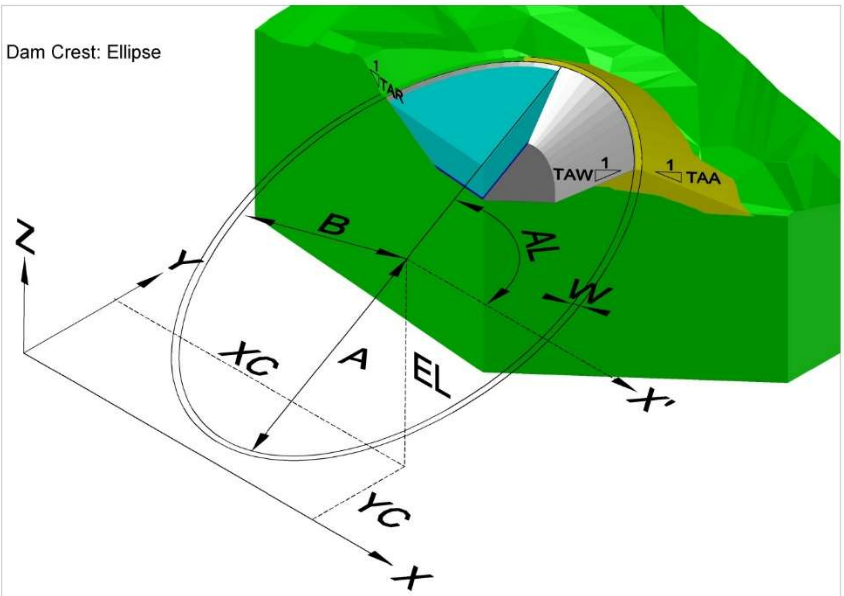


Figure 2-2: Notation - Elliptically Shaped Dam Crown

Legend:

A ... Ellipse semi-major axis length at crown
B ... Ellipse semi-minor axis length at crown
W ... Crown width
EL ... Crown elevation
TAR ... Tangent of rock slope (above crown)
TAA ... Tangent of dam slope at air face
TAW ... Tangent of dam slope as water face
XC ... X-coordinate of center of ellipse
YC ... Y-coordinate of center of ellipse
AL ... Rotation angle of ellipse

2.2.2 Arc Shaped Dam Crown

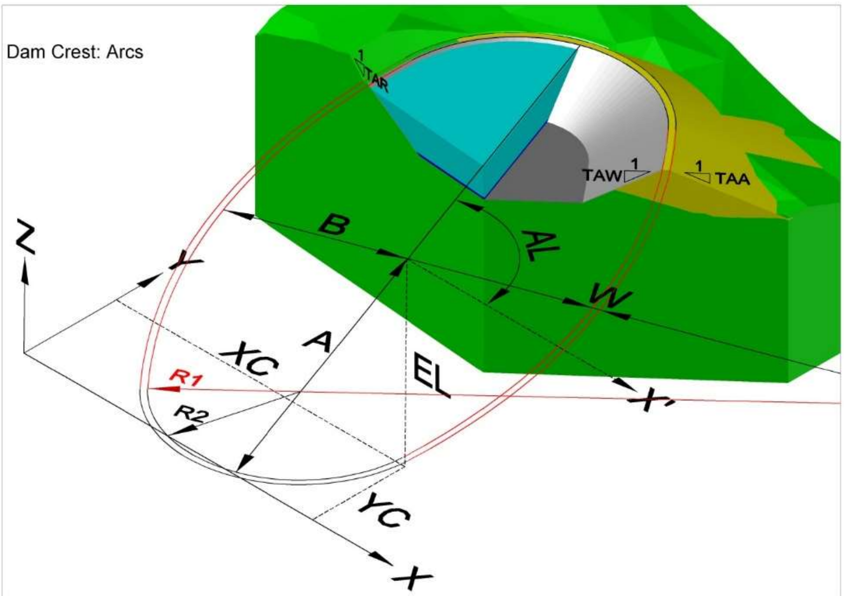


Figure 2-3: Notation - Arc Shaped Dam Crown

Legend:

A	...	Arc semi-major axis length at crown
B	...	Arc semi-minor axis length at crown
W	...	Crown width
EL	...	Crown elevation
TAR	...	Tangent of rock slope (above crown)
TAA	...	Tangent of dam slope at air face
TAW	...	Tangent of dam slope at water face
XC	...	X-coordinate of center of arcs
YC	...	Y-coordinate of center of arcs
AL	...	Rotation angle of arcs

2.2.3 Semi-circle Shaped Dam Crown

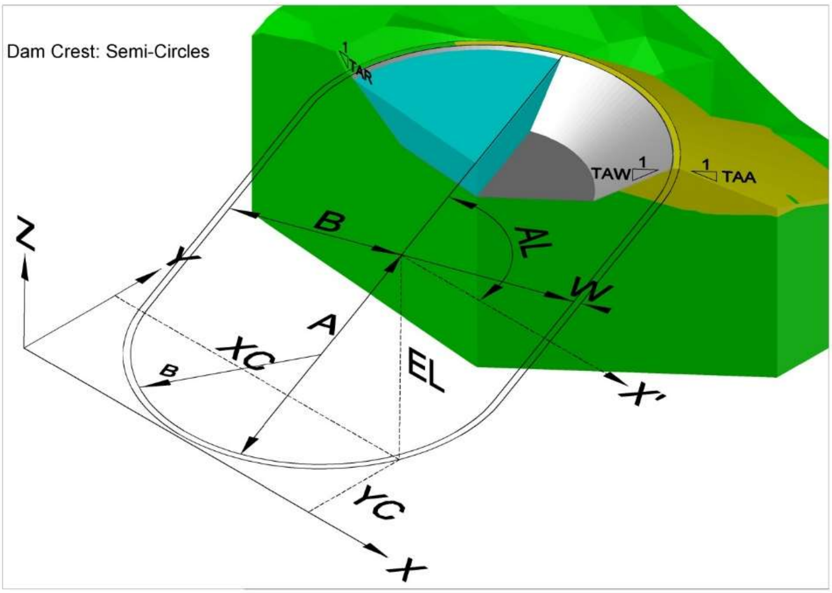


Figure 2-4: Notation - Semi-circle Shaped Dam Crown

Legend:

A	...	Semi-circle semi-major axis length at crown
B	...	Semi-circle semi-minor axis length at crown
W	...	Crown width
EL	...	Crown elevation
TAR	...	Tangent of rock slope (above crown)
TAA	...	Tangent of dam slope at air face
TAW	...	Tangent of dam slope at water face
XC	...	X-coordinate of center of semi-circles
YC	...	Y-coordinate of center of semi-circles
AL	...	Rotation angle of semi-circles

2.2.4 Polygonal Shaped Dam Crown – POLYLINE1

For type "POLYLINE1" only the inside crest polyline is defined. The outside crest polyline is at an off-set W from inside crest polyline. The inside crest polyline shall consist of lines and/or arcs.

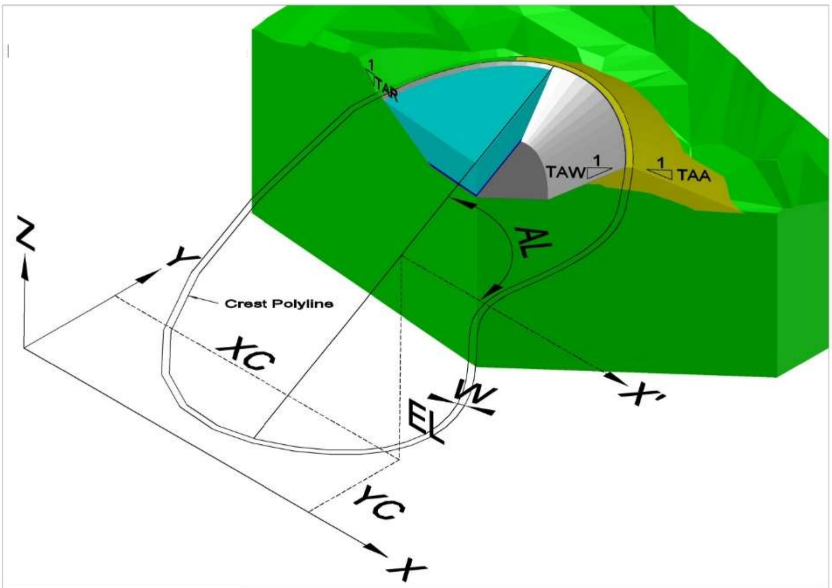


Figure 2-5: Notation - polygonal Shaped Dam Crown – Type POLYLINE1

Legend:

W ... Crown width
EL ... Crown elevation
TAR ... Tangent of rock slope (above crown)
TAA ... Tangent of dam slope at air face
TAW ... Tangent of dam slope at water face
XC ... X-coordinate of center of polyline
YC ... Y-coordinate of center of polyline
AL ... Rotation angle of polyline

2.2.5 Polygonal Shaped Dam Crown – POLYLINE2

For type "POLYLINE2" 2 polylines inside and outside crest polylines are defined. Inside and outside crest polylines shall consist of lines and/or arcs.

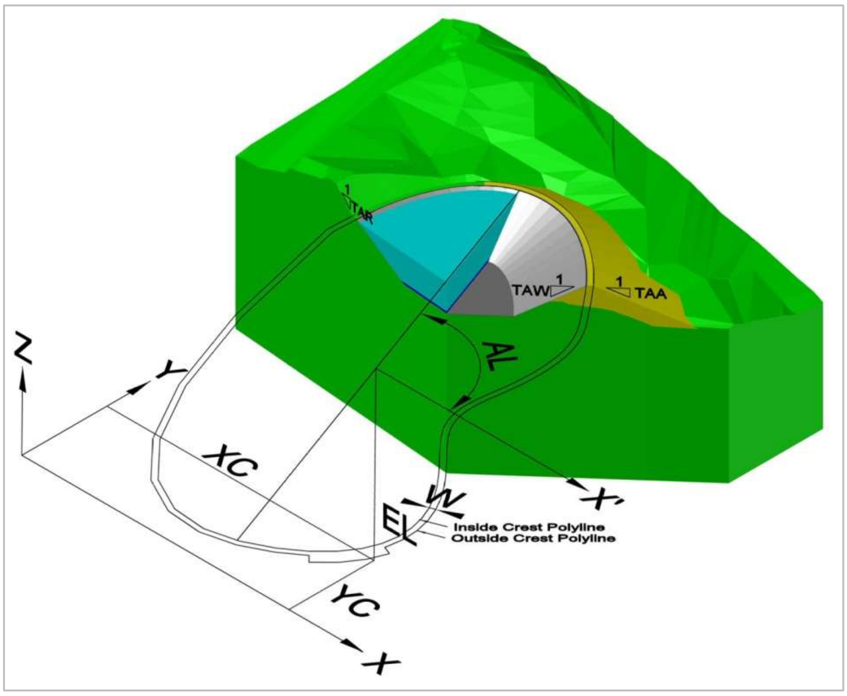


Figure 2-6: Notation - polygonal Shaped Dam Crown – Type POLYLINE2

Legend:

EL ... Crown elevation

TAR ... Tangent of rock slope (above crown)

TAA ... Tangent of dam slope at air face

TAW ... Tangent of dam slope at water face

XC ... X-coordinate of center of polyline

YC ... Y-coordinate of center of polyline

AL ... Rotation angle of polyline

3 RESERVOIR DAM MODULES

3.1 MAIN

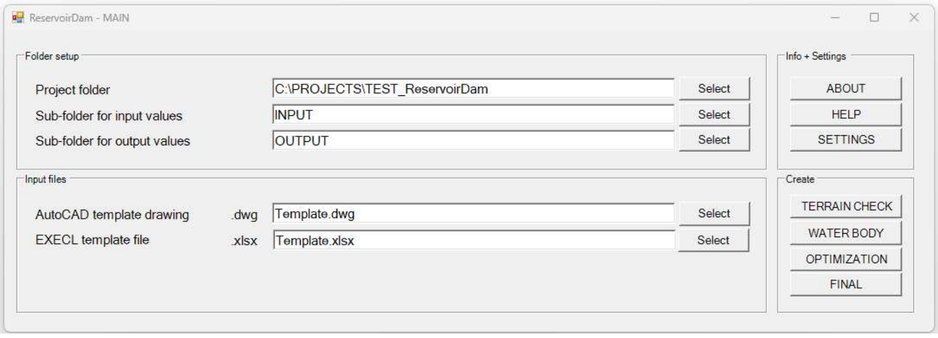


Figure 3-1: MAIN form

In the MAIN form the *input* and *output directory* as well as the *templates* are defined. The *template drawing* is an empty drawing with some settings.

Following files are delivered with the program ***ReservoirDam***:

- Template.dwg … empty AutoCAD drawing with some settings
- Template.xlsx … Excel file used for graphical presentation of the construction costs
- NORTH_ARROW.dxf … north indicator shown in the result plot files

3.2 ABOUT

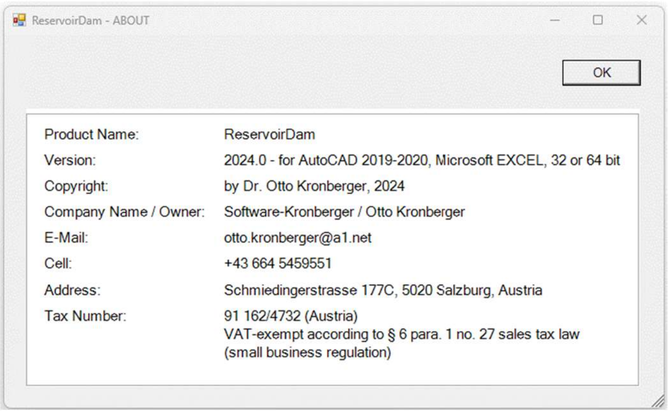


Figure 3-2: ABOUT form

3.3 SETTINGS

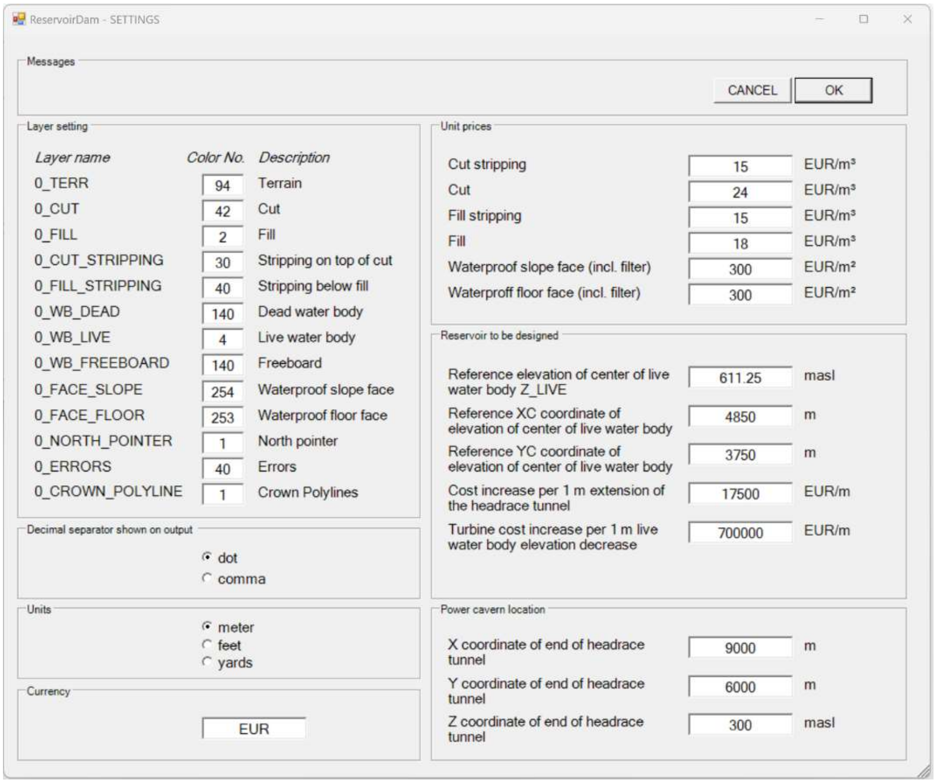


Figure 3-3: SETTINGS form

In the SETTINGS form following items can by defined:

- color number for layers
- decimal separator, which shall be shown on the output lists
- definition of units to be shown on the output lists
- unit prices and currency, etc.

Currency, unit prices, XC, YC, Z_LIVE, X, Y and Z values are used for POST-PROCESSING only.

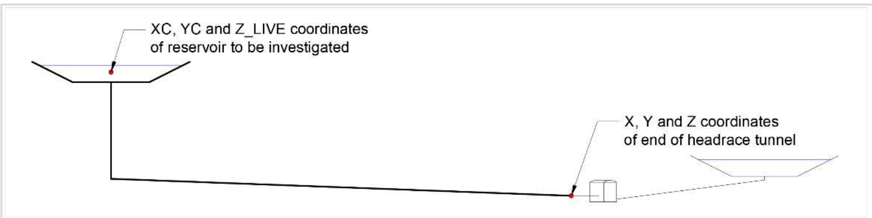


Figure 3-4: Notation - Cross section

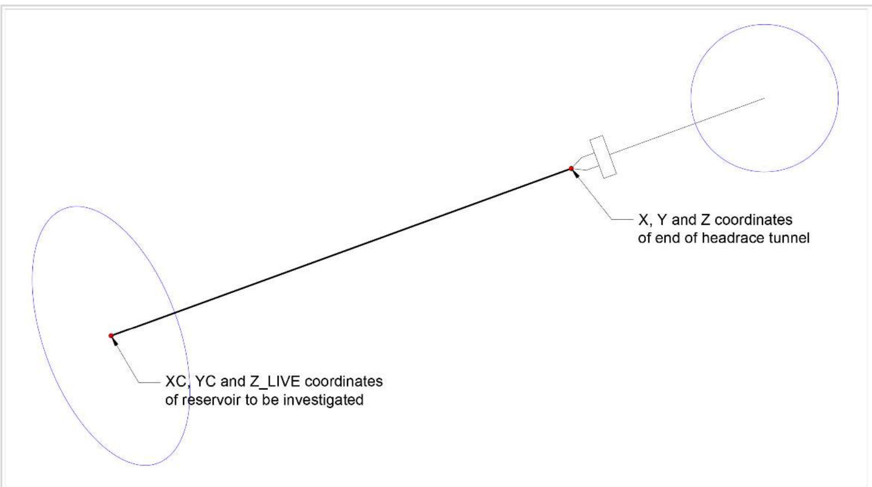


Figure 3-5: Notation - Plan view

3.4 TERRAIN CHECK (optionally)

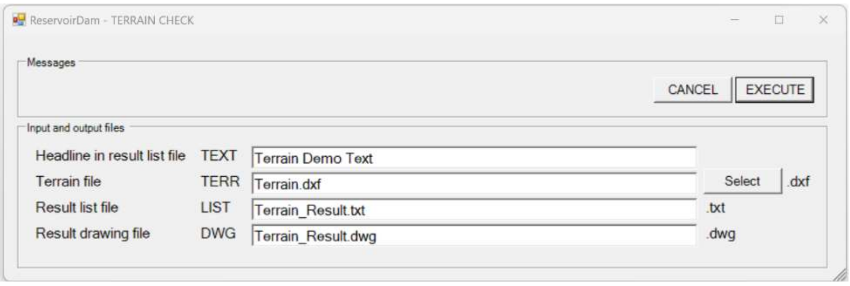


Figure 3-6: TERRAIN CHECK form

A terrain check is recommended in order to prevent AutoCAD from encountering fatal errors (mainly numerical errors) in OPTIMIZATION or FINAL module due to incompatible terrain elements.

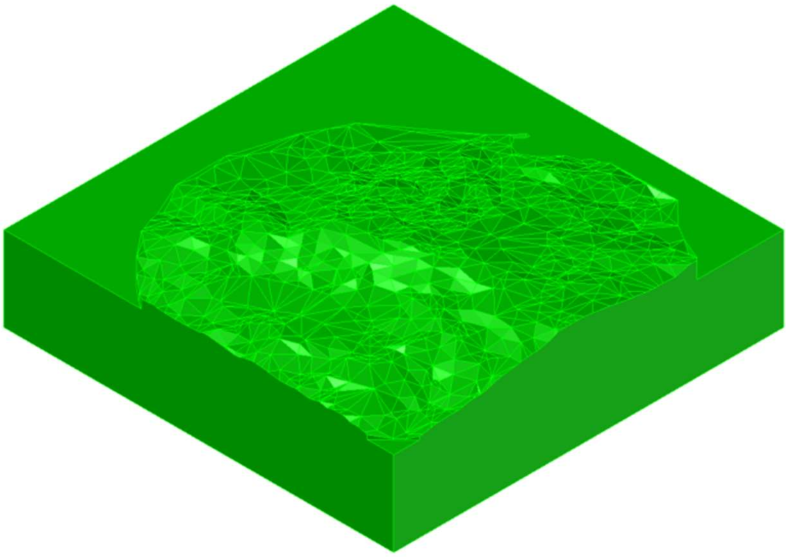


Figure 3-7: TERRAIN file (3D solid)

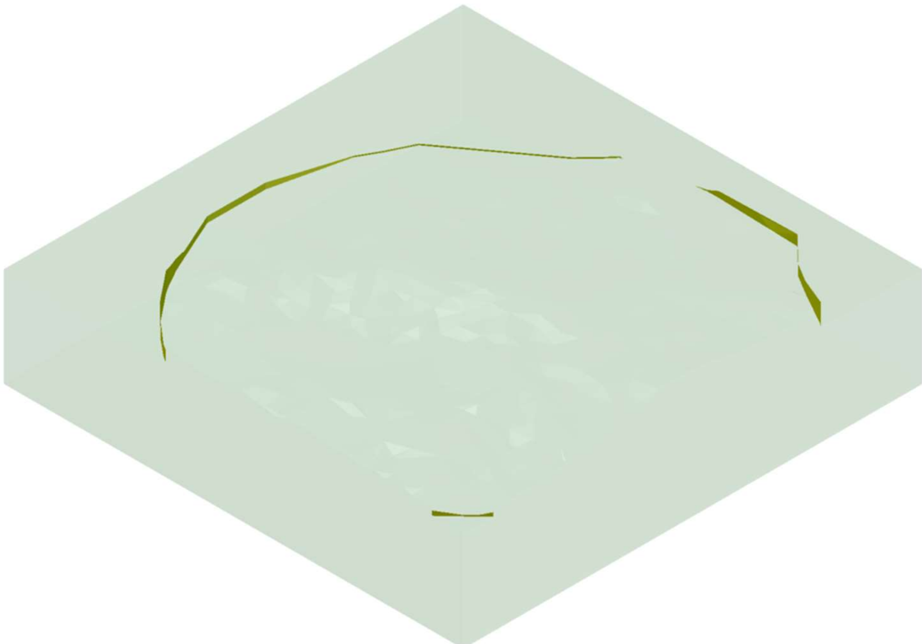


Figure 3-8: TERRAIN drawing file with Errors (red) and Warnings (yellow)

```
TERRAIN DEMO TEXT

   Terrain file                                            = C:\TEST_QuickDAM\INPUT\TERRAIN.dxf

   total number of elements                                =           1790
   No. of terrain 3DSolid floor regions                    =              1
   No. of terrain 3DSolid vertical boundary regions        =              4
   No. of terrain 3DSolid vertical boundary surfaces       =              0
   No. of terrain regions                                  =           1759
   No. of terrain surfaces                                 =              0
   No. of vertical terrain regions                         =             26  WARNINGS
   No. of vertical terrain surfaces                        =              0
   No. of overhanging terrain regions                      =              0
   No. of overhanging terrain surfaces                     =              0
   No. of regions that form holes in terrain body          =              0

   File showing errors (red) and warnings (yellow) in file = C:\TEST_QuickDAM\OUTPUT\TERRAIN_RESULT.dwg

   Notation:
   * regions  = plane elements
   * surfaces = curved elements
```

Figure 3-9: TERRAIN list file

3.5 WATER BODY (optionally)

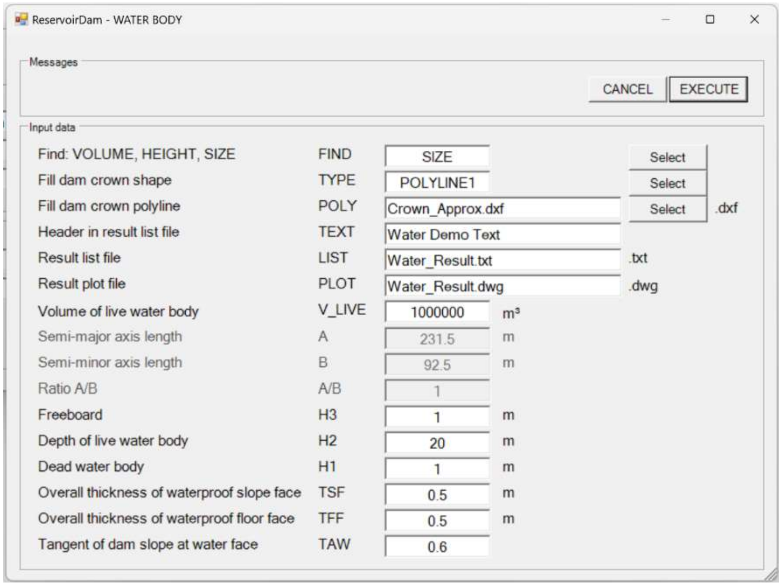


Figure 3-10: WATER BODY form

The WATER BODY module allows an initial estimation of

- volume of live water body V_LIVE
- or depth of live water body H2
- or size of dam crown (A and B, or scale factor for dam crown polyline)

without reference to the terrain.

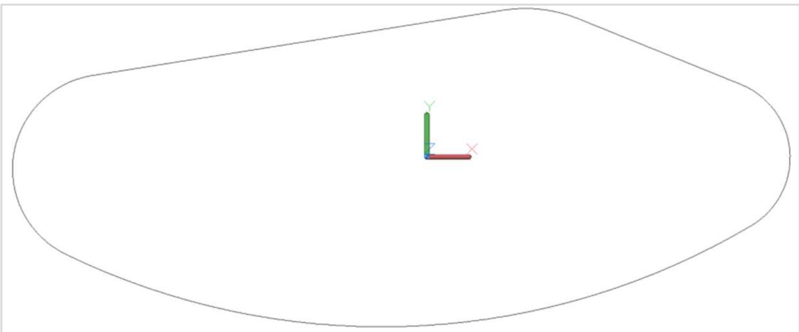


Figure 3-11: Crown Polyline

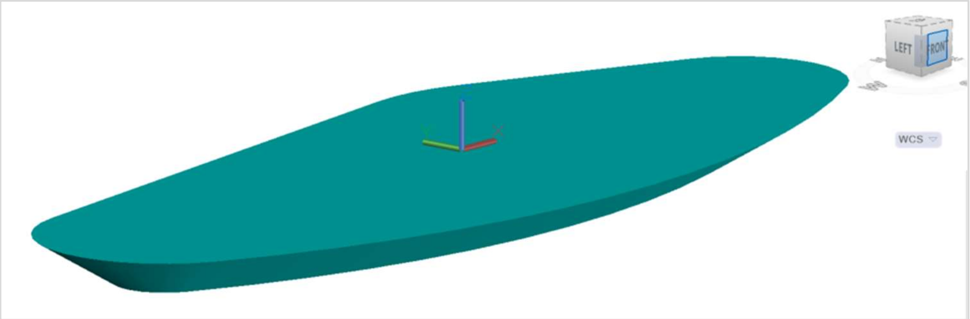


Figure 3-12: WATER BODY result plot file

```
WATER DEMO TEXT

INPUT
   File name of dam crest polyline             FILE    = C:\TEST_QuickDAM\INPUT\CROWN_1_APPROX.dxf
   Volume of live water body                   V_LIVE  =   1000000.000 m³
   Freeboard                                   H3      =         1.000 m
   Depth of live water body                    H2      =        20.000 m
   Depth of dead water body                    H1      =         1.000 m
   Thickness of waterproof slope face          TSF     =         0.500 m
   Thickness of waterproof floor face          TFF     =         0.500 m
   Ratio V/H of dam slope at water face        TAW     =         0.60000

RESULT
   Scale factor for crown polyline             SCALE   =    0.97062874
```

Figure 3-13: WATER-BODY result list file

3.6 OPTIMIZATION (optionally)

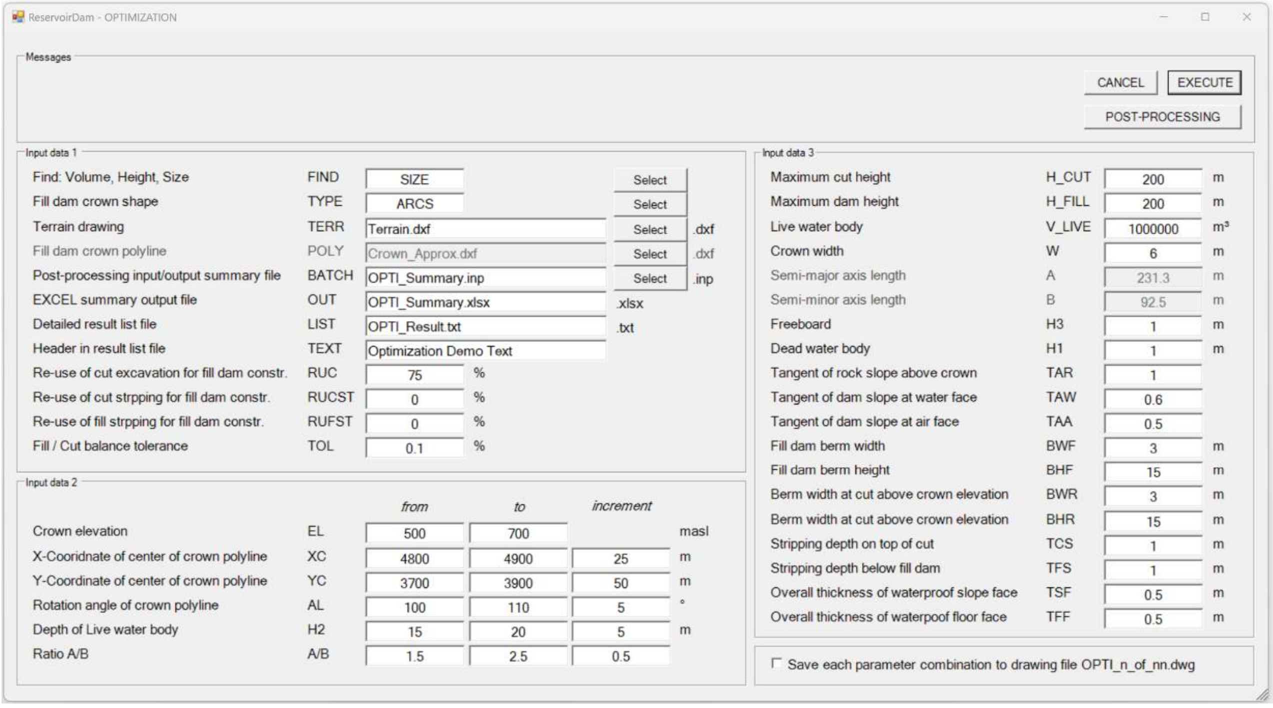


Figure 3-14: OPTIMIZATION form

For fill dam construction, it is usually necessary to re-use at least some of the excavation material, untreated or treated.

With the OPTIMIZATION EXECUTE module

- volume of live water body V_LIVE
- or depth of live water body H2
- or size of dam crown (A and B, or scale factor for dam crown polyline)

can be calculated fully automatically for numerous combinations such as

- X-coordinate of center of crown polyline XC
- Y-coordinate of center of crown polyline YC

- rotation angle of crown polyline AL

and the corresponding volumes, areas, lengths and especially crown elevation are calculated. The crown elevation is calculated so that the specified $Ratio = \frac{\text{reusable cut,cut stripping and fill stripping}}{fill}$ is maintained.

A 3D drawing and a corresponding entry in the list files (*Post-processing input/output summary file* and *Detailed result list file*) are created for each combination.

```
Combination;XC;YC;AL;SCALE;A;B;A/B;H2;EL;V_LIVE;Z_LIVE;V_CSTR;V_CUT;V_FSTR;V_FILL;A_SLOPE;A_FLOOR;ERROR
1 of 450;4800.000;3700.000;100.000;1.000;195.679;130.453;1.500;15.000;599.609;999999.041;591.555;75962.164;1463939.311;84914.276;1098672.993;32228.355;53673.391;0;
2 of 450;4800.000;3700.000;105.000;1.000;195.679;130.453;1.500;15.000;599.609;999999.041;591.555;76054.135;1472870.740;85746.460;1105371.481;32228.355;53673.391;0;
3 of 450;4800.000;3700.000;110.000;1.000;195.679;130.453;1.500;15.000;599.463;999999.041;591.408;76183.035;1486729.018;86191.668;1115147.690;32228.355;53673.391;0;
4 of 450;4800.000;3750.000;100.000;1.000;195.679;130.453;1.500;15.000;601.440;999999.041;593.386;76604.340;1386425.388;82076.090;1040524.515;32228.355;53673.391;0;
5 of 450;4800.000;3750.000;105.000;1.000;195.679;130.453;1.500;15.000;601.367;999999.041;593.313;76678.134;1396741.497;82281.465;1047860.300;32228.355;53673.391;0;
6 of 450;4800.000;3750.000;110.000;1.000;195.679;130.453;1.500;15.000;601.111;999999.041;593.056;76834.434;1414079.171;82712.693;1059896.900;32228.355;53673.391;0;
7 of 450;4800.000;3800.000;100.000;1.000;195.679;130.453;1.500;15.000;602.747;999999.041;594.692;74572.822;1304977.023;87505.159;978109.402;32228.355;53673.391;0;
8 of 450;4800.000;3800.000;105.000;1.000;195.679;130.453;1.500;15.000;602.832;999999.041;594.777;74539.614;1300819.520;86830.637;976316.405;32228.355;53673.391;0;
9 of 450;4800.000;3800.000;110.000;1.000;195.679;130.453;1.500;15.000;602.734;999999.041;594.680;74617.046;1303986.185;86109.414;978209.180;32228.355;53673.391;0;
```

Figure 3-15: OPTIMIZATION – Post-processing input/output summary file

```
--------------------------------------------------------------------------
Parameter Combination 1 of 450

   Arc semi-major axis length                     A       =     195.679 m
   Arc semi-minor axis length                     B       =     130.453 m
   Elevation of crown                             EL      =     599.609 masl
   Width of crown                                 W       =       6.000 m
   Freeboard                                      H3      =       1.000 m
   Depth of live water body                       H2      =      15.000 m
   Depth of dead water body                       H1      =       1.000 m
   Berm width  at rock cut above crown            BWR     =       3.000 m
   Berm height at rock cut above crown            BHR     =      15.000 m
   Berm width  at fill dam                        BWF     =       3.000 m
   Berm height at fill dam                        BHF     =      15.000 m
   Ratio V/H of rock slope above dam crown        TAR     =       1.000
   Ratio V/H of dam slope at water face           TAW     =       0.600
   Ratio V/H of dam slope at air face             TAA     =       0.500
   Stripping depth below fill dam                 TFS     =       1.000 m
   Stripping depth on top of cut                  TCS     =       1.000 m
   Thickness of waterproof slope face             TSF     =       0.500 m
   Thickness of waterproof floor face             TFF     =       0.500 m
   X-coordinate of center of crest polyline       XC      =    4800.000 m
   Y-coordinate of center of crest polyline       YC      =    3700.000 m
   Rotation angle of crest polyline               AL      =     100.000 °

   Fill versus excavation balance                 BAL     =       1.001
   Elevation of floor of basin                    Z_FLOOR =     582.109 masl
   Volume of freeboard water body                 V_FREE  =   79730.528 m³
   Volume of live water body                      V_LIVE  =  999999.041 m³
   Volume of dead water body                      V_DEAD  =   54393.893 m³
   Elevation of center of live water body         Z_LIVE  =     591.555 masl
   Volume of stripping on top of cut              V_CSTR  =   75962.164 m³
   Volume of cut                                  V_CUT   = 1463939.311 m³
   Volume of fill                                 V_FILL  = 1098672.993 m³
   Volume of stripping below fill dam             V_FSTR  =   84914.276 m³
   Volume of waterproof slope face                V_SLOPE =   16166.101 m³
   Volume of waterproof floor face                V_FLOOR =   26657.661 m³
   Area of waterproof slope face                  A_SLOPE =   32228.355 m²
   Area of waterproof floor face                  A_FLOOR =   53673.391 m²
--------------------------------------------------------------------------
Parameter Combination 2 of 450

   Arc semi-major axis length                     A       =     195.679 m
   Arc semi-minor axis length                     B       =     130.453 m
   Elevation of crown                             EL      =     599.609 masl
   Width of crown                                 W       =       6.000 m
   Freeboard                                      H3      =       1.000 m
   Depth of live water body                       H2      =      15.000 m
   Depth of dead water body                       H1      =       1.000 m
```

Figure 3-16: OPTIMINZATION - Detailed result list file

With the OPTIMIZATION POST-PROCESSING module the most cost-effective parameter combination can be determined.

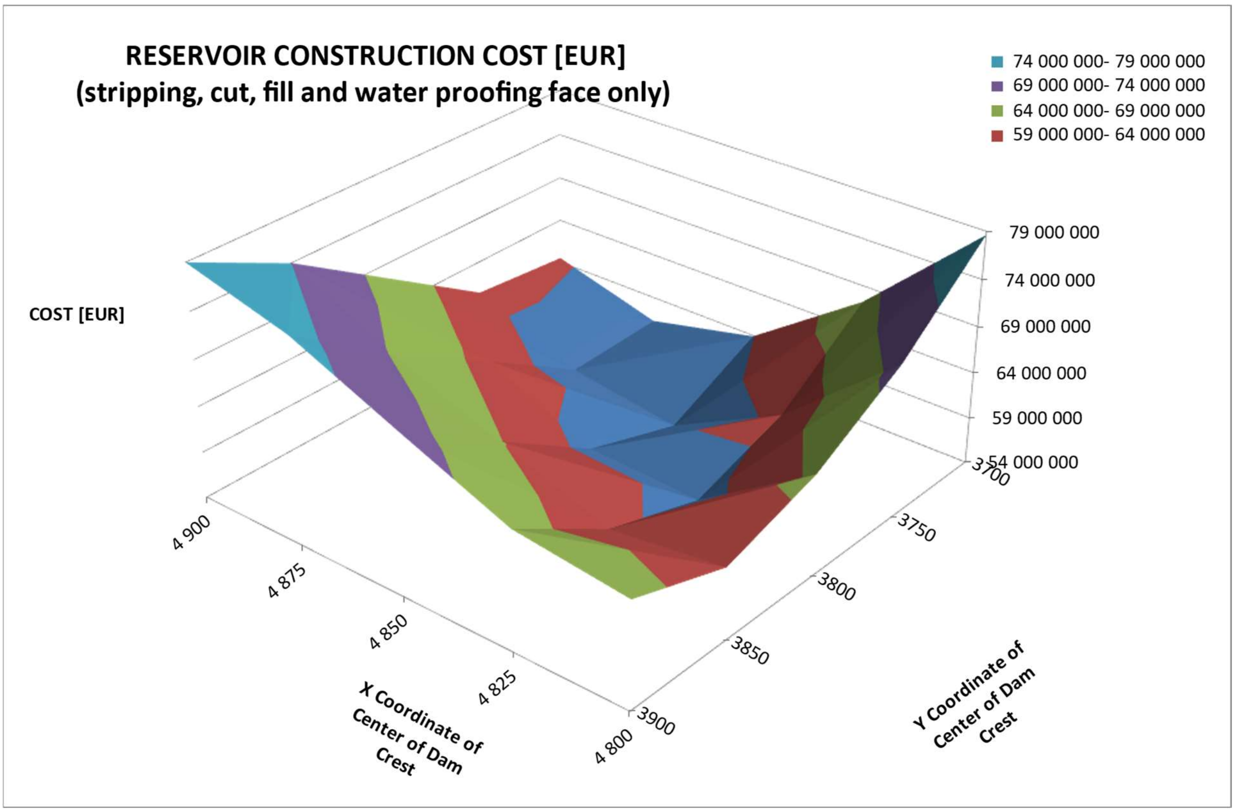


Figure 3-17: OPTIMINZATION – EXCEL summary output file chart

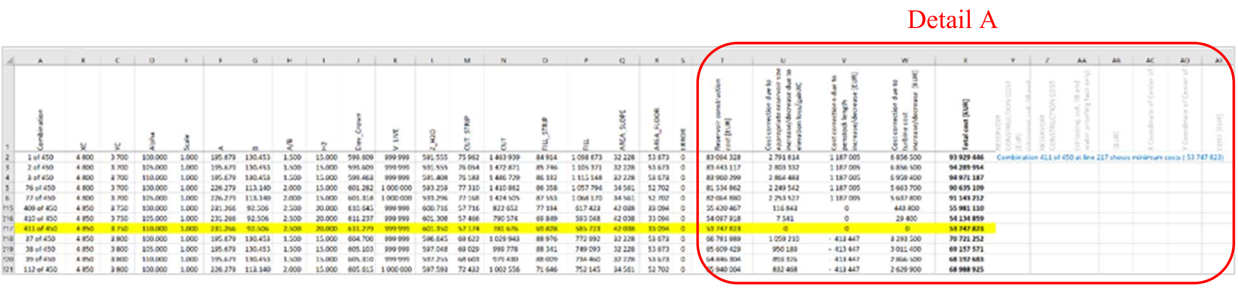
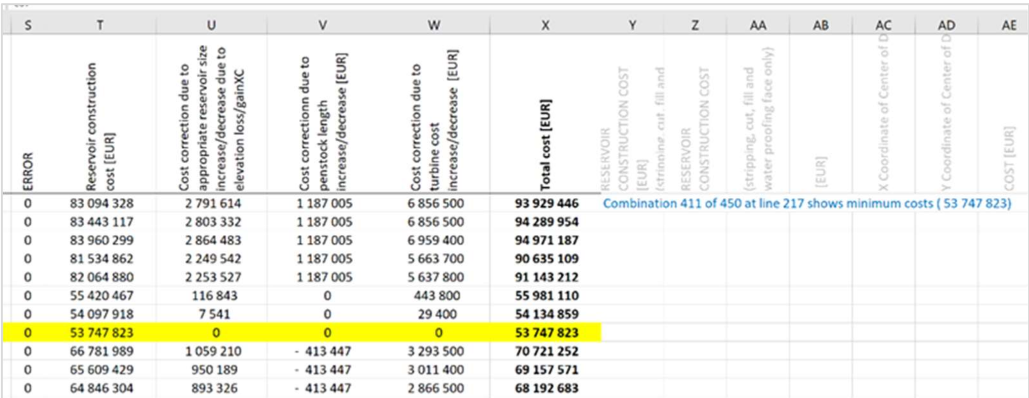


Figure 3-18: OPTIMINZATION – EXCEL summary output file



S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
ERROR	Reservoir construction cost [EUR]	Cost correction due to appropriate reservoir size increase/decrease due to elevation loss/gainXC	Cost correctionn due to penstock length increase/decrease [EUR]	Cost correction due to turbine cost increase/decrease [EUR]	Total cost [EUR]	RESERVOIR CONSTRUCTION COST [EUR] (stripping, cut, fill and	RESERVOIR CONSTRUCTION COST	(stripping, cut, fill and water proofing face only)	[EUR]	X Coordinate of Center of D	Y Coordinate of Center of D	COST [EUR]
0	83 094 328	2 791 614	1 187 005	6 856 500	93 929 446	Combination 411 of 450 at line 217 shows minimum costs (53 747 823)						
0	83 443 117	2 803 332	1 187 005	6 856 500	94 289 954							
0	83 960 299	2 864 483	1 187 005	6 959 400	94 971 187							
0	81 534 862	2 249 542	1 187 005	5 663 700	90 635 109							
0	82 064 880	2 253 527	1 187 005	5 637 800	91 143 212							
0	55 420 467	116 843	0	443 800	55 981 110							
0	54 097 918	7 541	0	29 400	54 134 859							
0	53 747 823	0	0	0	53 747 823							
0	66 781 989	1 059 210	- 413 447	3 293 500	70 721 252							
0	65 609 429	950 189	- 413 447	3 011 400	69 157 571							
0	64 846 304	893 326	- 413 447	2 866 500	68 192 683							

Figure 3-19: OPTIMINZATION – EXCEL summary output file – Detail A

3.7 FINAL

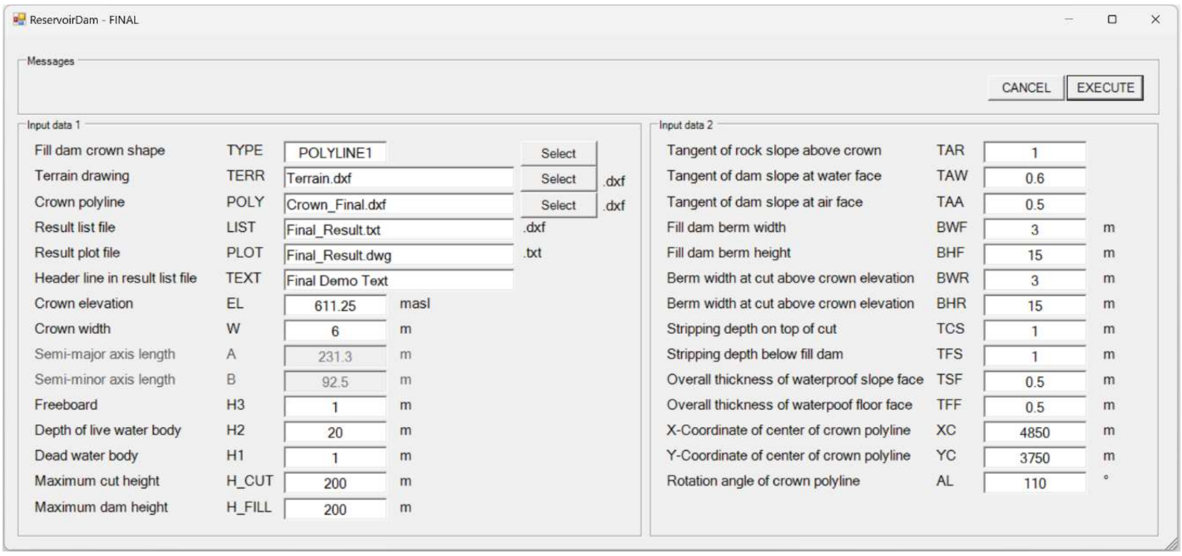


Figure 3-20: FINAL form

With the FINAL module, the reservoir dam is created both as a 3D drawing and as a list file according to the values shown in Figure 3-20.

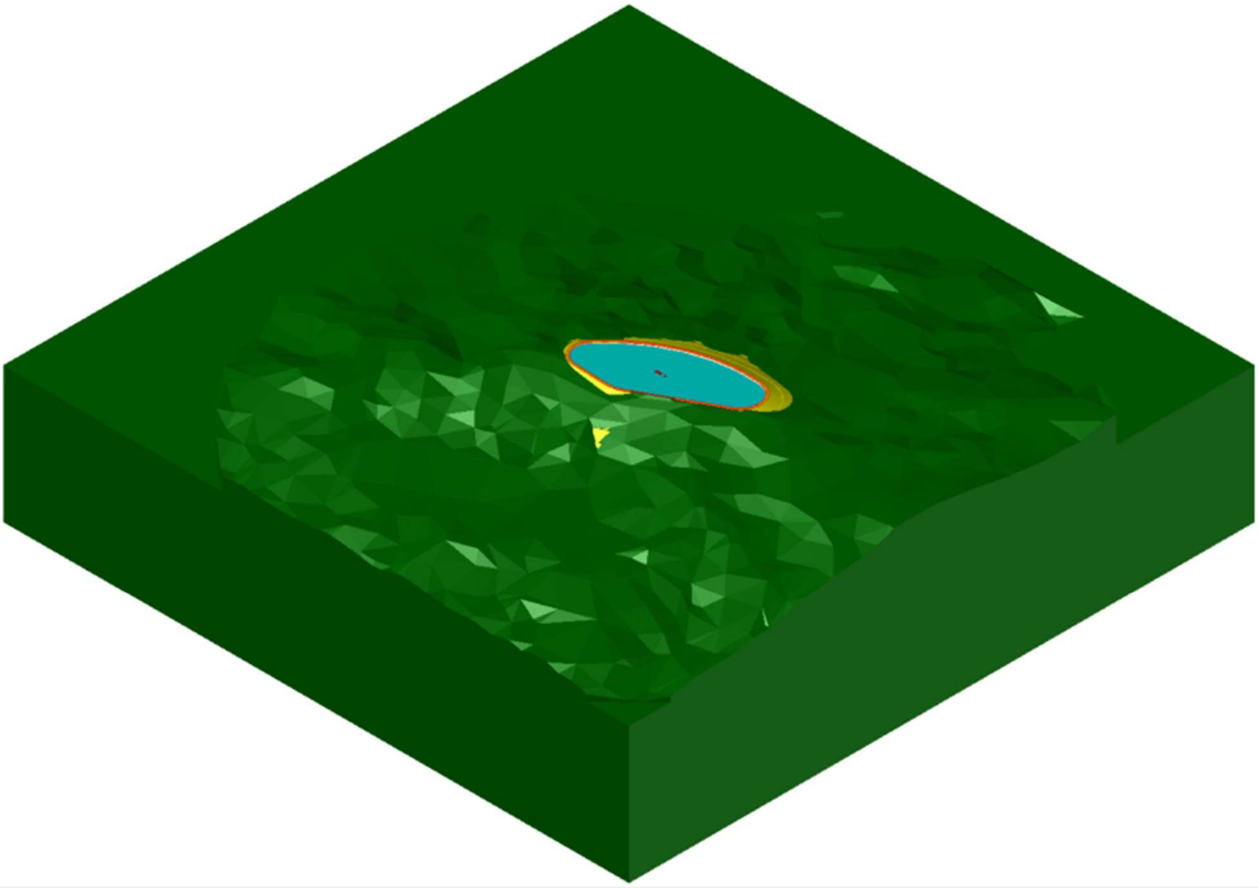


Figure 3-21: FINAL – Result plot file

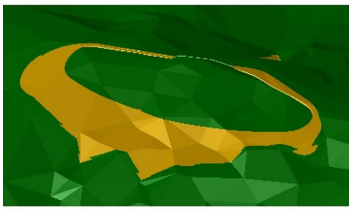
```
FINAL DEMO TEXT

INPUT
   File name of dam crest polyline                  FILE    = C:\TEST_QuickDAM\INPUT\CROWN_1_FINAL.dxf
   Elevation of crown                               EL      =       610.250 masl
   Width of crown                                   W       =         6.000 m
   Freeboard                                        H3      =         1.000 m
   Depth of live water body                         H2      =        20.000 m
   Depth of dead water body                         H1      =         1.000 m
   Berm width  at rock cut above crown              BWR     =         3.000 m
   Berm height at rock cut above crown              BHR     =        15.000 m
   Berm width  at fill dam                          BWF     =         3.000 m
   Berm height at fill dam                          BHF     =        15.000 m
   Ratio V/H of rock slope above dam crown          TAR     =         1.000
   Ratio V/H of dam slope at water face             TAW     =         0.600
   Ratio V/H of dam slope at air face               TAA     =         0.500
   Stripping depth below fill dam                   TFS     =         1.000 m
   Stripping depth on top of cut                    TCS     =         1.000 m
   Thickness of waterproof slope face               TSF     =         0.500 m
   Thickness of waterproof floor face               TFF     =         0.500 m
   X-coordinate of center of crest polyline         XC      =      4850.000 m
   Y-coordinate of center of crest polyline         YC      =      3750.000 m
   Rotation angle of crest polyline                 AL      =       110.000 °

RESULT
   Elevation of floor of basin                      Z_FLOOR =       587.750 masl
   Volume of freeboard water body                   V_FREE  =     67901.452 m³
   Volume of live water body                        V_LIVE  =   1000000.235 m³
   Volume of dead water body                        V_DEAD  =     33445.397 m³
   Elevation of center of live water body           Z_LIVE  =       600.344 masl
   Volume of stripping on top of cut                V_CSTR  =     58571.514 m³
   Volume of cut                                    V_CUT   =    776705.367 m³
   Volume of fill                                   V_FILL  =    585246.351 m³
   Volume of stripping below fill dam               V_FSTR  =     60621.866 m³
   Volume of waterproof slope face                  V_SLOPE =     21533.895 m³
   Volume of waterproof floor face                  V_FLOOR =     16177.009 m³
   Area of waterproof slope face                    A_SLOPE =     42934.363 m²
   Area of waterproof floor face                    A_FLOOR =     32718.230 m²
   Perimeter of waterproof slope face at crown      P_CROWN =      1099.567 m
   Perimeter of waterproof floor face               P_FLOOR =       869.184 m
   Maximum height of fill dam                       H_FILL  =        67.231 m
   Maximum height of rock cut above crown elev.     H_CUT   =         8.750 m
```

Figure 3-22: FINAL – Result list file

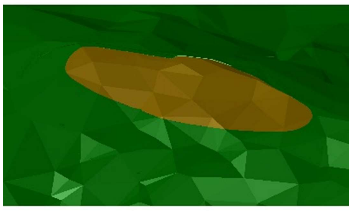
CONSTRUCTION STAGES:

Origin Terrain

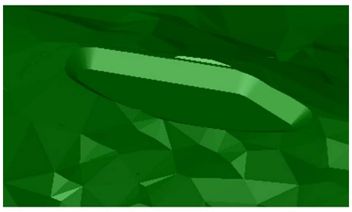
Excavation of cut-stripping



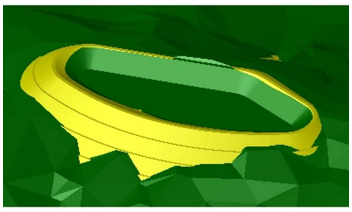
Excavation of fill stripping



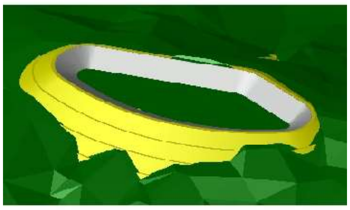
Excavation of cut



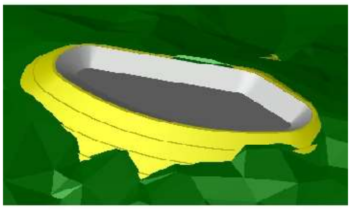
Excavation completed



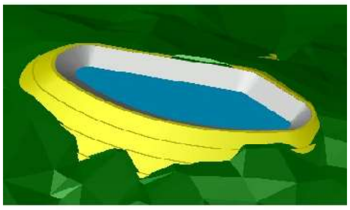
Construction of fill dam



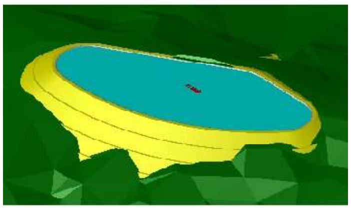
Construction of water-proof slope face



Construction of water-proof floor face



Completed reservoir with dead water body



Completed reservoir with live water body

4 PROGRAM VERIFICATION

4.1 General

For verification of the *ReservoirDam* software a circular shaped reservoir as shown in Figure 4-1 below is used. A horizontal terrain surface at elevation +400 is assumed.

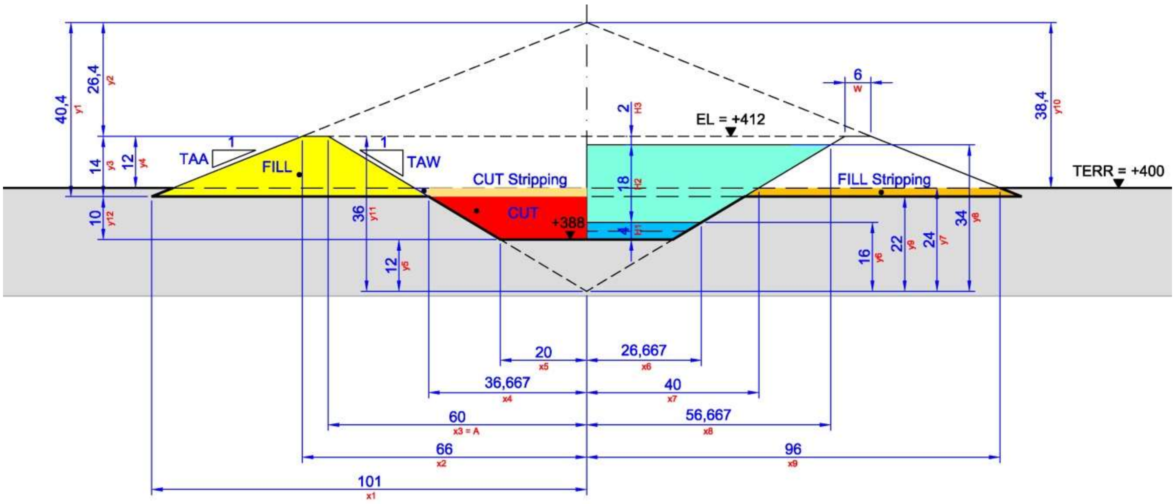


Figure 4-1: Program verification – Notation for circular reservoir

4.2 ReservoirDam Results

```
INPUT
   Arc semi-major axis length                      A        =        60.000 m
   Arc semi-minor axis length                      B        =        60.000 m
   Elevation of crown                              EL       =       412.000 masl
   Width of crown                                  W        =         6.000 m
   Freeboard                                       H3       =         2.000 m
   Depth of live water body                        H2       =        18.000 m
   Depth of dead water body                        H1       =         4.000 m
   Berm width  at rock cut above crown             BWR      =         0.000 m
   Berm height at rock cut above crown             BHR      =         0.000 m
   Berm width  at fill dam                         BWF      =         0.000 m
   Berm height at fill dam                         BHF      =         0.000 m
   Ratio V/H of rock slope above dam crown         TAR      =         1.000
   Ratio V/H of dam slope at water face            TAW      =         0.600
   Ratio V/H of dam slope at air face              TAA      =         0.400
   Stripping depth below fill dam                  TFS      =         2.000 m
   Stripping depth on top of cut                   TCS      =         2.000 m
   Thickness of waterproof slope face              TSF      =         0.000 m
   Thickness of waterproof floor face              TFF      =         0.000 m
   X-coordinate of center of crest polyline        XC       =         0.000 m
   Y-coordinate of center of crest polyline        YC       =         0.000 m
   Rotation angle of crest polyline                AL       =         0.000 °

RESULT
   Elevation of floor of basin                     Z_FLOOR  =       388.000 masl
   Volume of freeboard water body                  V_FREE   =     21386.101 m³
   Volume of live water body                       V_LIVE   =    102415.921 m³
   Volume of dead water body                       V_DEAD   =      6888.233 m³
   Elevation of center of live water body          Z_LIVE   =       403.071 masl
   Volume of stripping on top of cut               V_CSTR   =      9238.610 m³
   Volume of cut                                   V_CUT    =     25947.228 m³
   Volume of fill                                  V_FILL   =    206402.405 m³
   Volume of stripping below fill dam              V_FSTR   =     54856.164 m³  see note 1
   Volume of waterproof slope face                 V_SLOPE  =         0.000 m³
   Volume of waterproof floor face                 V_FLOOR  =         0.000 m³
   Area of waterproof slope face                   A_SLOPE  =     11723.824 m²
   Area of waterproof floor face                   A_FLOOR  =      1256.637 m²
   Perimeter of waterproof slope face at crown     P_CROWN  =       376.991 m
   Perimeter of waterproof floor face              P_FLOOR  =       125.664 m
   Maximum height of fill dam                      H_FILL   =        14.000 m
   Maximum height of rock cut above crown elev.    H_CUT    =         0.000 m
```

Figure 4-2: Program verification – ReservoirDam results

Note 1: The volume V_FSTR shows a slightly higher value in chapter 4.2 than in 4.3. The reason for this is that in chapter 3 the fill stripping ends vertically, but in chapter 4 it is overhanging. Please compare Figure 2-1 and Figure 4-1.

4.3 Analytical Results

For notation, please see Figure 4-1

INPUT

W := 6	x1 := 101	y1 := 40.4
H3 := 2	x2 := 66	y2 := 26.4
H2 := 18	x3 := 60	y3 := 14
H1 := 4	x4 := 36.666667	y4 := 12
TAW := 0.6	x5 := 20	y5 := 12
TAA := 0.4	x6 := 26.666667	y6 := 16
	x7 := 40	y7 := 24
A := x3	x8 := 56.666667	y8 := 34
B := x3	x9 := 96	y9 := 22
		y10 := 38.4
		y11 := 36

RESULTS

$V\_fill\_str := x1^2 \cdot \pi \cdot \frac{y1}{3} - x9^2 \cdot \pi \cdot \frac{y10}{3} - x7^2 \cdot \pi \cdot \frac{y7}{3} + x4^2 \cdot \pi \cdot \frac{y9}{3}$	V_fill_str = 51736	m3
$V\_cut\_str := x7^2 \cdot \pi \cdot \frac{y7}{3} - x4^2 \cdot \pi \cdot \frac{y9}{3}$	V_cut_str = 9239	m3
$V\_cut := x4^2 \cdot \pi \cdot \frac{y9}{3} - x5^2 \cdot \pi \cdot \frac{y5}{3}$	V_cut = 25947	m3
$V\_fill := x1^2 \cdot \pi \cdot \frac{y1}{3} - x2^2 \cdot \pi \cdot \frac{y2}{3} - x3^2 \cdot \pi \cdot \frac{y11}{3} + x4^2 \cdot \pi \cdot \frac{y9}{3}$	V_fill = 206402	m3
$V\_free := x3^2 \cdot \pi \cdot \frac{y11}{3} - x8^2 \cdot \pi \cdot \frac{y8}{3}$	V_free = 21386	m3
$V\_live := x8^2 \cdot \pi \cdot \frac{y8}{3} - x6^2 \cdot \pi \cdot \frac{y6}{3}$	V_live = 102416	m3
$V\_dead := x6^2 \cdot \pi \cdot \frac{y6}{3} - x5^2 \cdot \pi \cdot \frac{y5}{3}$	V_dead = 6888	m3
$A\_floor := x5^2 \cdot \pi$	A_floor = 1257	m2
$A\_slope := \frac{x3 + x5}{2} \cdot 2 \cdot \pi \cdot \frac{H1 + H2 + H3}{\sin(\mathrm{atan}(TAW))}$	A_slope = 11724	m2

Figure 4-3: Program verification – Analytical results

4.4 Conclusion

ReservoirDam results are in accordance with analytical results.